7 Concentrations and Gradients of Stress and Strain

Notch effects have been a key problem in the study of fatigue for more than 125 years, since Wohler showed that adding material to a railway axle might make it weaker in fatigue. He stated that the radius at the shoulder between a smaller and a larger diameter is of prime importance to the fatigue life of axles and that fatigue cracks will start at the transition from a smaller to a larger section.

Notches cannot be avoided in many structures and machines. A bolt has notches in the thread roots and at the transition between the head and the shank. Rivet holes in sheets, welds on plates, and keyways on shafts are all notches. Although notches can be very dangerous, they can often be rendered harmless by suitable treatment.

To understand the effects of notches and the means to overcome them, one must consider five parameters in addition to the behavior of smooth specimens of the same material:

- Concentrations of stress and of strain
- Stress gradients
- Mean stress effects and residual stresses
- Local yielding
- Nucleation and growth of cracks

In some cases, one of the five parameters by itself may explain the difference in behavior between a smooth part and a notched part that has an equal cross section at the root of the notch. Even when several parameters are involved, it is often possible to use "variable constants" or "notch factors" that correlate the test results. We intend to avoid the use of adjustable factors and must therefore consider the effects of all five parameters. In the end, this will be less difficult than the adjustment of a single notch factor.

Notches concentrate stresses and strains. The degree of concentration is a factor in the fatigue strength of notched parts. It is measured by the elastic stress concentration factor, K_t, defined as the ratio of the maximum stress, σ, or strain, ε, at the notch to the nominal stress, S, or strain:

$$K_t = \frac{\sigma}{S} = \frac{\varepsilon}{e} \text{ as long as } \frac{\sigma}{\varepsilon} = const = E$$

where σ and ε *e* represent local stress and strain at the notch and *S* and *e* represent nominal stress and strain, respectively. Let us consider a sheet with a central circular hole. K_t depends on the ratio of hole diameter to sheet width. Figure 9.1 shows K_t plotted versus the ratio of hole diameter to sheet width. Two curves are shown. In the upper curve the nominal stress is defined as load divided by total or gross area (w·*t)*. In the lower curve the nominal stress is defined as load divided by net area, i.e., the area remaining after the hole has been cut out. In this book we use the net area to define the nominal stress when using stress concentration factors unless otherwise stated. However, in calculating the stress intensity factor from the nominal stress, we use the gross area as if the crack did not exist.

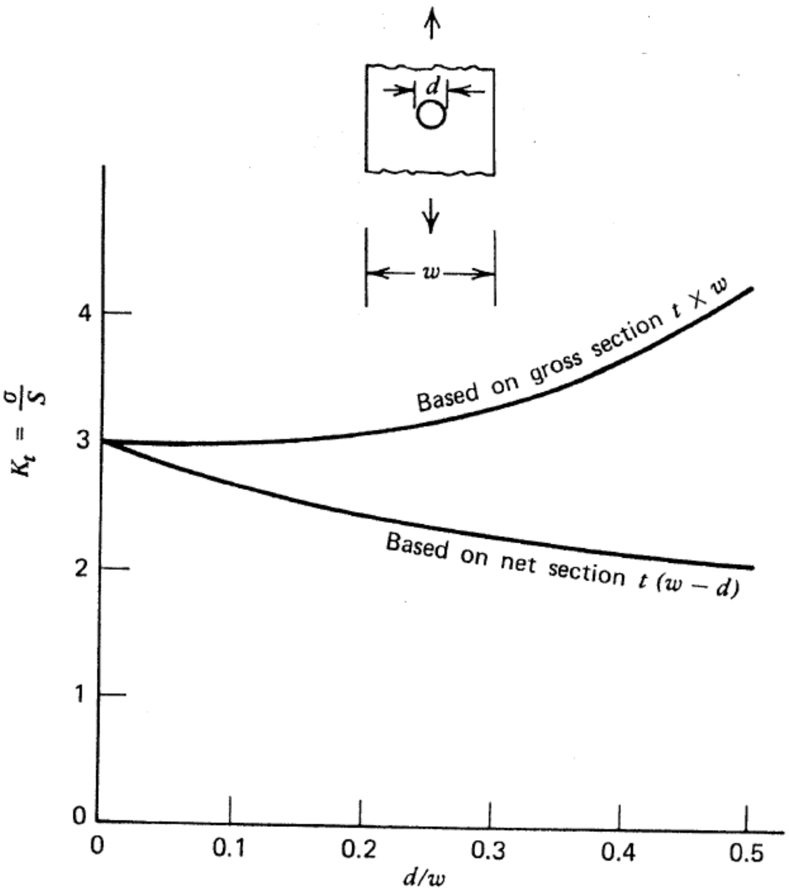


Fig. 9.1 Elastic stress concentration factor for a central circular hole in a sheet

Figure 9.2 shows the stresses near a circular hole in the center of a wide sheet in tension. This problem was investigated and solved as early as 1898 by Kirsch [1]. The following equations for the axial stress, σ_y, and the transverse stress, σ_x, on a transverse line through the center of the hole for plane stress are taken from

$$\frac{\sigma_y}{S} = 1 + 0.5\left(\frac{r}{x}\right)^2 + 1.5\left(\frac{r}{x}\right)^4;$$

$$\frac{\sigma_x}{S} = 1.5\left(\frac{r}{x}\right)^2 - 1.5\left(\frac{r}{x}\right)^4,$$

where S = nominal stress = load/area,

σ_y = axial stress,

σ_x = transverse stress,

x = distance from center of hole,

r = radius of hole.

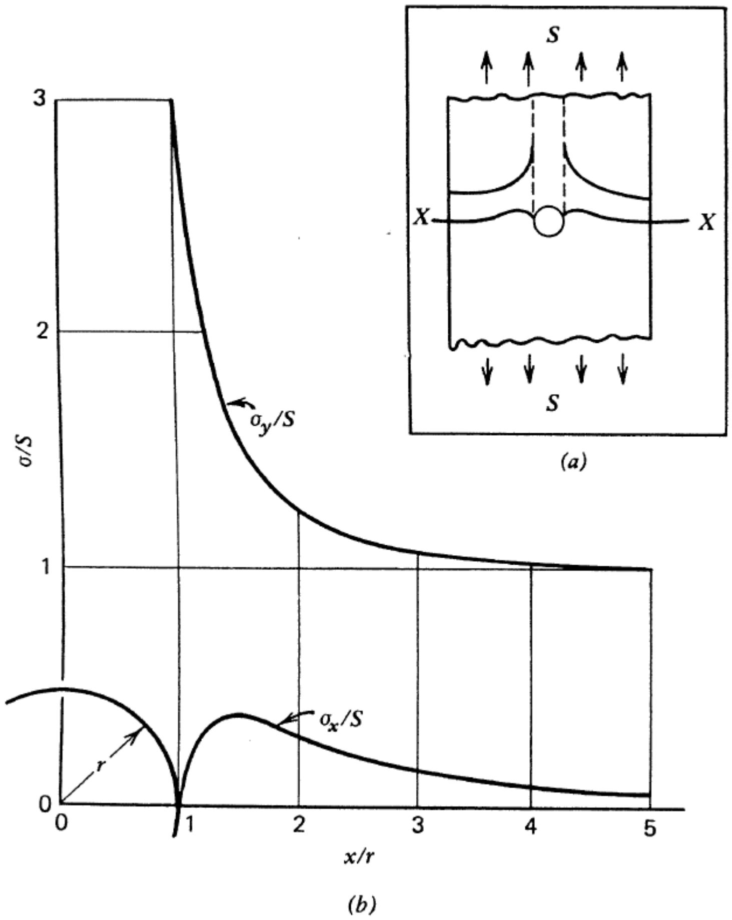


Fig. 9.2 Stress distribution along the section X-X near a hole in a wide sheet

Values of σ_y/S and σ_x/S are plotted versus x/r in Fig. 9.2. We see that σ_y/S decreases quite rapidly as the distance from the edge of the hole is increased. K_t at the edge of the hole is 3, while at a distance of 0.25r the value of σ_y/S is only about 2. At a distance of $2r$ it is only 1.07. In other words, the stress at the edge of the hole is three times the nominal stress, but at a distance from the hole edge equal to the diameter, it is only about 7 percent higher than the nominal stress. Also, the state of stress at the notch is uniaxial and away from the notch it is biaxial for this plane stress condition.

The slope of the σ_y versus x curve at the edge of the hole is another measure of the rapid decrease in stress as we move away from the edge of the hole. The rapid decrease in stress with increasing distance from the notch and the existence of biaxial or triaxial states of stress at a small distance from the notch are typical of stress concentrations. They explain why we cannot expect to predict the behavior of notched parts with great accuracy by applying stress concentration factors to the fatigue strength values obtained from smooth parts. Numerical values of stress gradients or simple design formulas for stress distribution near notches are not often readily available in the literature.

It is important to know where the most dangerous stress concentrations exist in a part. Charts of stress concentration factors are available in the literature. Examples of such charts for stepped shafts in tension, bending, and torsion are shown in Fig. 9.3, and for a plate with opposite U-shaped notches in tension and bending are shown in Fig. 9.4. It is important, however, to remember that elastic stress concentration factors for homogeneous isotropic materials depend only on geometry (independent of material) and mode of loading, and that they apply only when the notch is in the linear elastic deformation condition.

For qualitative estimates, some engineers like to use an imperfect analogy between stresses or strains and liquid flow. Restrictions or enlargements in a pipe produce local increases in flow velocity somewhat similar to the local increases in stresses produced by changes in cross section. The designer will try to “streamline” the contours of parts, as indicated in Fig. 9.5. Consider, for instance, an elliptic hole in a wide sheet. Placed lengthwise with the forces or flow, it produces less stress concentration and less flow interference than when it is placed crosswise.

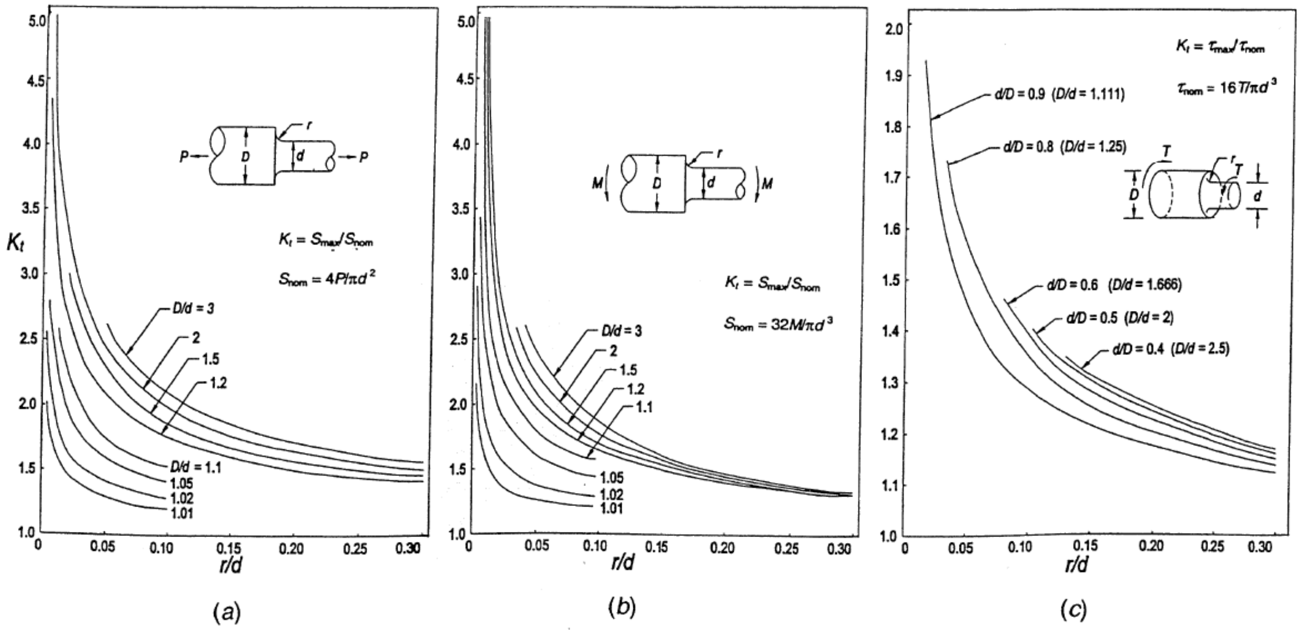


Fig 9.3 Stress concentrations factors for a stepped shaft (a)Tension, (b)Bending, (c)Torsion

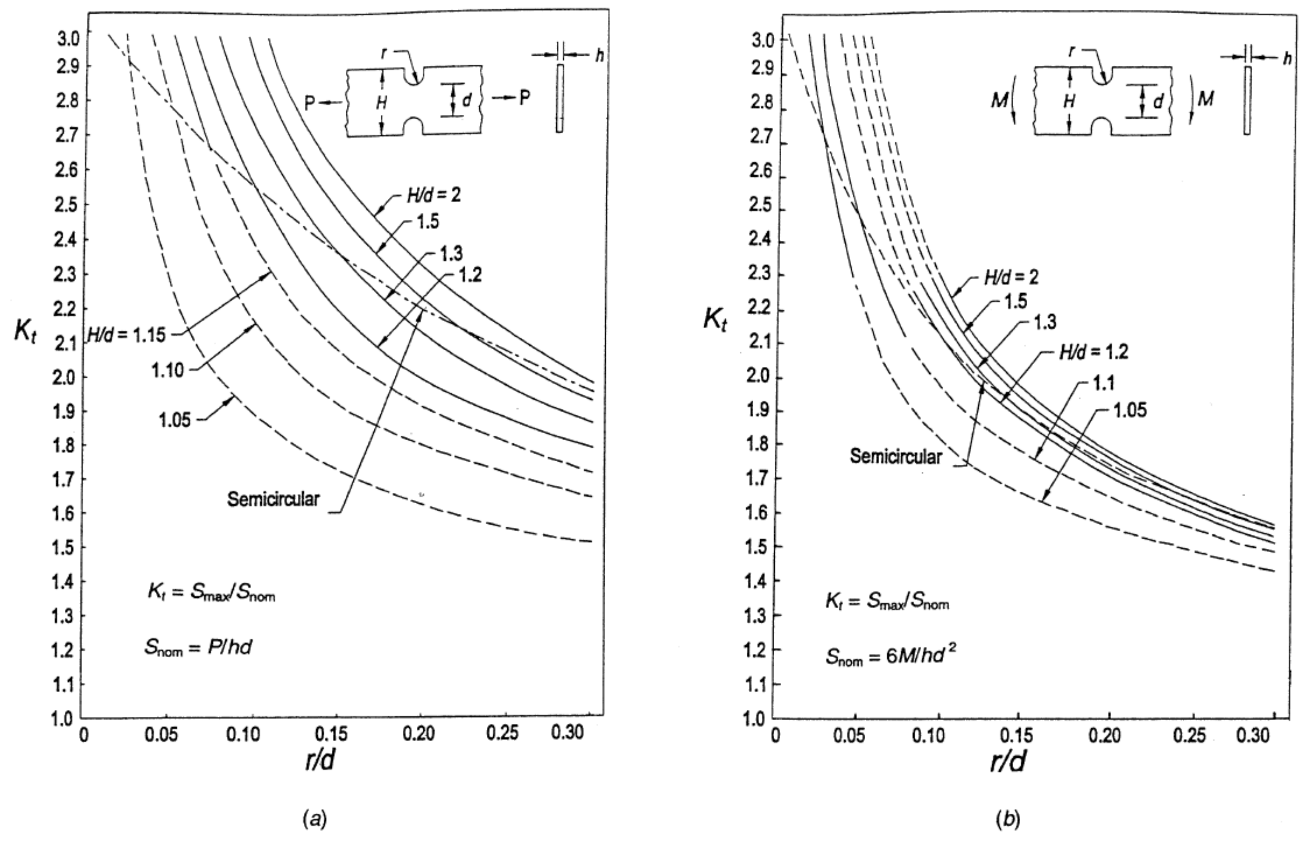


Fig 9.4 Stress concentrations factors for a flat bar with opposite U-shaped notches (a)Tension, (b)Bending

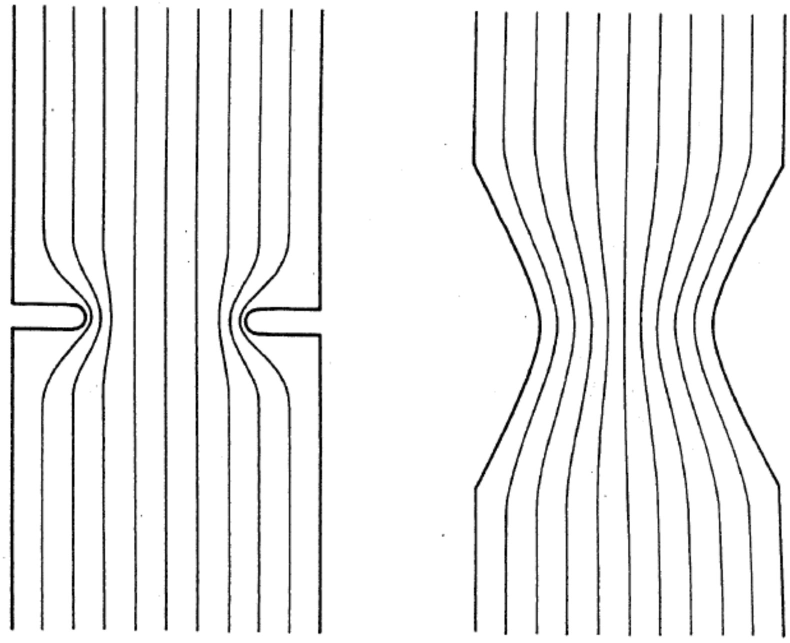
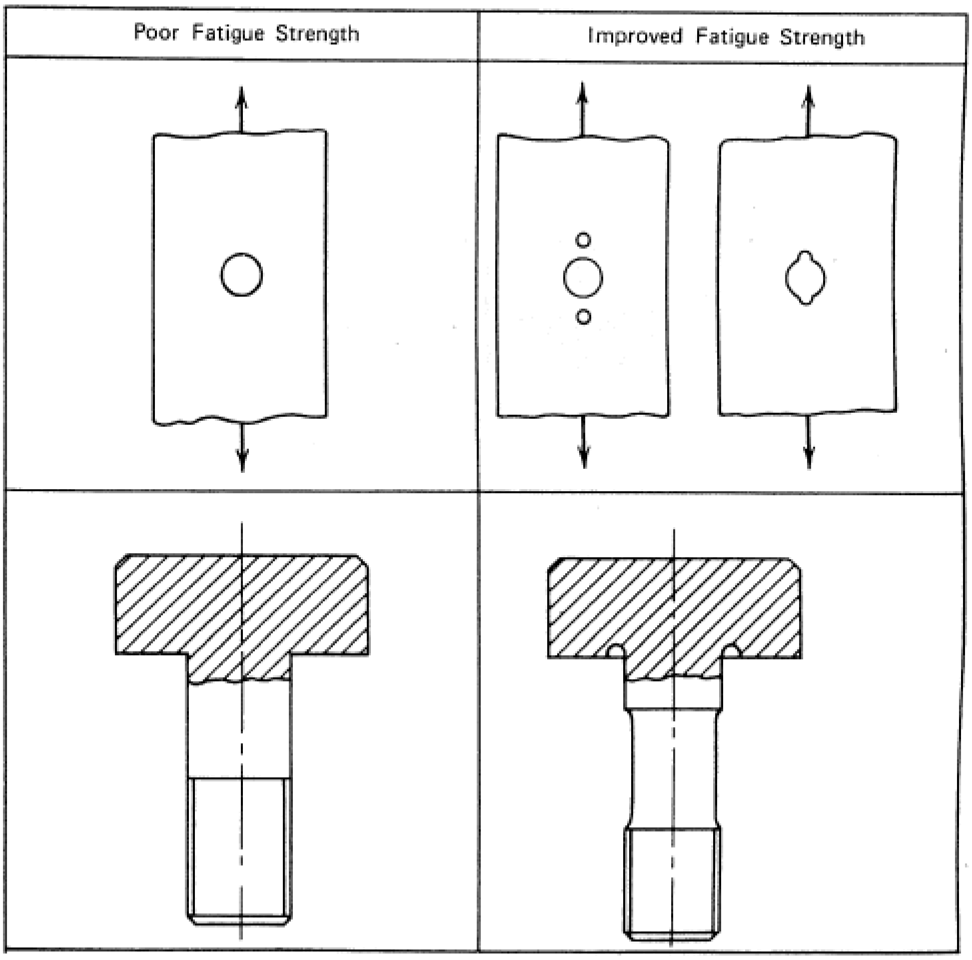
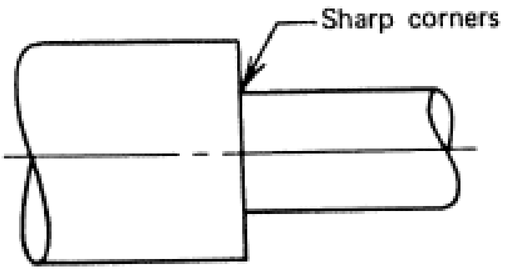
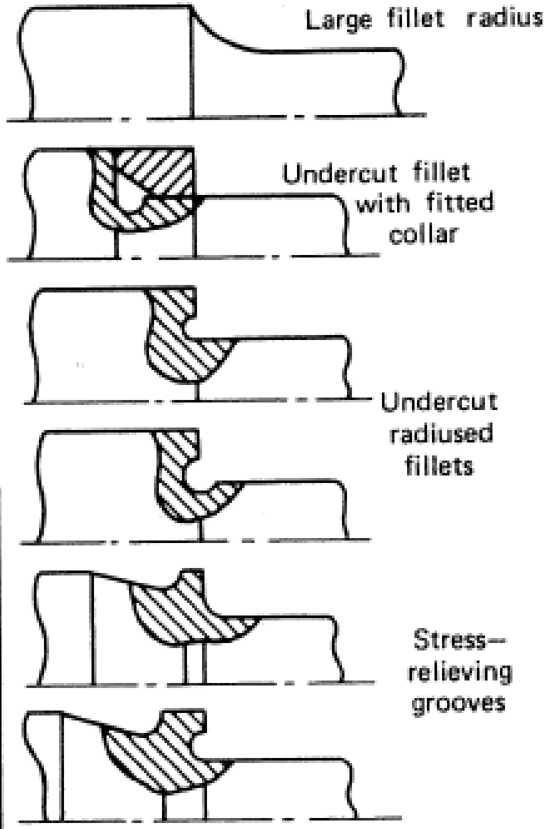
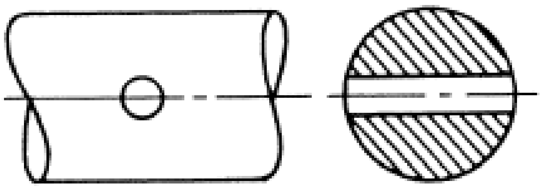
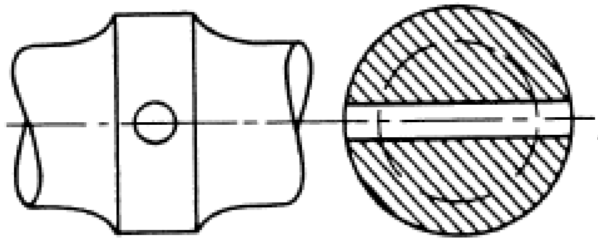
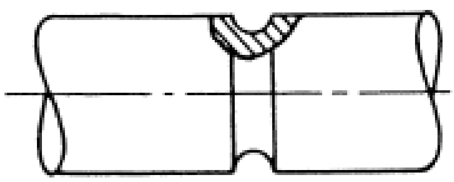
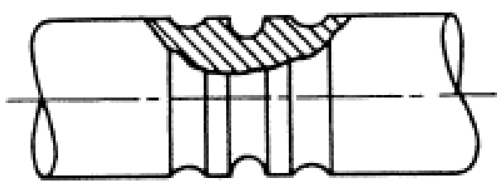
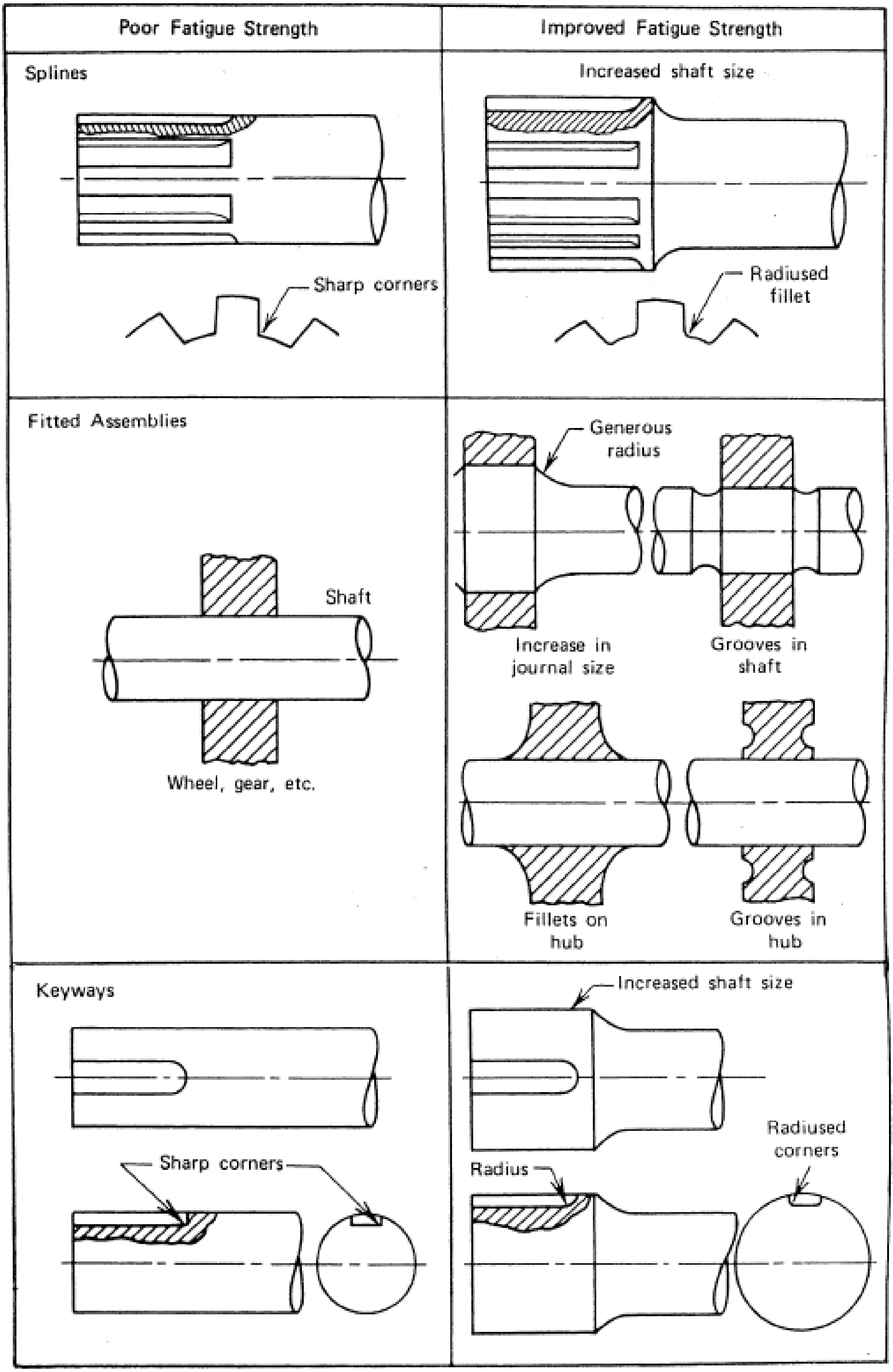


Fig. 9.5 The crowding and bending of flow lines near obstructions helps to visualize the concentration of stresses and strains near notches. The large section and the small section are the same in both cases, but the transitions are different



Poor Fatigue Strength	Improved Fatigue Strength
Shoulders 	
Holes 	Enlarged section at hole  Stress–relieving grooves
Grooves 	Stress–relieving grooves 


Poor Fatigue Strength
Improved Fatigue Strength
Splines
Sharp corners
Increased shaft size
Radiused fillet
Fitted Assemblies
Shaft
Wheel, gear, etc.
Generous radius
Increase in journal size
Grooves in shaft
Fillets on hub
Grooves in hub
Keyways
Sharp corners
Increased shaft size
Radius
Radiused corners